Comorbid Word Reading and Mathematics Computation Difficulty at Start of First Grade

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Abstract

The purpose of this analysis was to describe cognitive processes associated with comorbid difficulty between word reading (WR) and mathematics computation (MC) at the start of first grade among children selected for WR and MC delays. A sample of 234 children (mean age 6.50 years, $SD = 0.31$) was assessed on WR, MC, core cognitive processes (phonological processing, rapid automatized naming, verbal counting [VC]), and domain-general cognitive processes (working memory, oral language, nonverbal reasoning, attentive behavior). Structural equation modeling was used to predict a latent Comorbidity factor, which modeled shared variance between WR and MC, and to identify processes associated with that Comorbidity factor. Results identified each of the core cognitive processes, especially VC, and each of the domain-general cognitive processes, especially working memory, as explaining shared variance between WR and MC. Implications for understanding comorbid difficulty at the start of first grade and designing coordinated first-grade interventions are discussed.

Keywords

word reading, mathematics computation, comorbid difficulty, comorbid learning disabilities

Word reading (WR) and mathematics computation (MC) typically develop in a parallel fashion (e.g., Ehri, 2005; Geary, 1993). In kindergarten, with respect to WR, most children form a reliable set of associations between visual and phonological representations of single letters as well as rudimentary word knowledge based largely on grapho-semantic connections. Kindergarten WR development parallels kindergarten mathematics development. This includes developing a reliable set of associations among visual, phonological, and semantic representations of numerals, basic insights into addition and subtraction concepts, and skill counting objects to solve problems with small numbers.

At first grade, parallels between reading and mathematics involve children's consolidation of efficient procedures. In reading, children link written whole words with phonological representations through decoding; in mathematics, they consolidate the most-efficient counting strategies to derive answers to arithmetic problems. These procedural strategies for linking phonological representations with words and for linking arithmetic problems with answers repeatedly produce correct associations. Repeated associations help establish words and arithmetic problems in memory, which in turn permits reliance on direct retrieval instead

of procedural strategies for reading words and solving arithmetic problems.

Later in first grade, children rely on connections at the unit of letter sequence blends, morphemes, and syllables. This accelerates the rate of consolidating new words. This developmental process relies in part on orthographic mapping (e.g., Ehri, 2014), in which printed words and phonics patterns are stored in long-term memory. This permits efficient retrieval of known words and efficient decoding and consolidation of similarly structured words. In MC, a parallel process concurrently develops, in which math facts accumulate in long-term memory. This process, along with children's increasingly sophisticated number knowledge (e.g., inverse relation of subtraction and addition; commutative property of

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addition), facilitates recognition of patterns that support efficient retrieval of known and related math facts as well as the application of decomposition strategies for problem-solving. (For learning standards at the end of kindergarten and first grade in the country where this study's data were collected, see Supplemental File.)

Unfortunately, not all children achieve kindergarten developmental milestones in WR and MC or demonstrate the expected first-grade learning trajectories required for long-term success. The focus of the present study is children with delays in WR and MC at the start of first grade. Such delays signal a risk for poor developmental trajectories in WR and MC. Comorbid difficulty across WR and MC occurs frequently (Landerl & Moll, 2010; Mann Koepke & Miller, 2013). Half of children with low performance in one domain experience low performance in the other (Koponen et al., 2018), and the risk for such comorbidity is two to five times greater than expected by chance (Martin & Fuchs, 2022).

Comorbid WR and MC difficulty also creates serious vulnerabilities. For example, WR skill is foundational to text comprehension (Catts et al., 2005; Fuchs et al., 2015), and MC skill is foundational to word problems (Fuchs et al., 2006), fractions (Jordan et al., 2013), and algebra (Fuchs et al., 2012; Tolar et al., 2009). Additionally, compared with peers with difficulty in a single area, students with comorbid difficulty experience weaker outcomes in each domain as well as less adequate response to generally effective intervention (Fuchs, Fuchs, & Prentice, 2004; Fuchs et al., 2013; Willcutt et al., 2013). Moreover, reading achievement and mathematics achievement are predictors of quality of life, financial security, and life expectancy (Batty et al., 2010; Ritchie & Bates, 2013).

Despite the frequency, severity, and seriousness of WR and calculations comorbidity, the dominant focus in the literature on early academic development and difficulty involves one domain or the other. The purpose of the present analysis was to describe cognitive processes that are associated with comorbid difficulty at the start of first grade. The hope is that understanding shared cognitive processes associated with comorbid difficulty at the time when formal schooling on WR and MC gains momentum will provide insights for designing interventions to address students' WR and MC needs in a coordinated fashion.

Prior Research

The cognitive processes investigated in prior work on predictors of WR and MC development and within developmental psychology more generally are frequently discussed within two categories. *Domain-general cognitive processes* (e.g., working memory) support learning in many different domains. This includes supporting the development of *core cognitive processes* (e.g., phonological processing [PP]),

which involve stimuli that are more proximal to the predicted learning targets. The direction of effects from domain-general to core cognitive processes (rather than the reverse) reflects the following: The development of core cognitive processes, which typically emerge in children during preschool, relies on domain-general cognitive abilities present at younger ages (Chu et al., 2016).

In terms of WR and MC outcomes, prior research focuses on fluency or accuracy. *Fluency* refers to reading words or solving calculation problems quickly and with accuracy (i.e., with little conscious attention to the mechanics of reading [e.g., decoding] or calculating [e.g., counting]). Fluency assessment is timed such that few children complete all items. Accuracy assessment is designed to provide sufficient time for most children to complete all items, such that children may rely on effort-demanding procedures as needed.

In considering prior research, we begin by summarizing five studies, representative of the literature that simultaneously modeled cognitive pathways to WR and MC in primary-grade children. Koponen et al. (2007) predicted 178 Finnish kindergarteners' fourth-grade single-digit addition and multiplication fluency, computation accuracy, and text-reading fluency. Predictors included Draw-A-Man, listening comprehension, processing speed, rapid automatized naming (RAN), verbal counting (VC), letter naming accuracy, and mother's education. With all variables in the model, RAN and VC were unique predictors of single-digit MC fluency and text-reading fluency; number knowledge and mother's education were unique predictors of computation accuracy after controlling for single-digit MC fluency. Koponen et al. (2013) provided corroborating evidence for the role of RAN and VC in WR and MC fluency. These studies suggest that RAN and PP predict fluency but not accuracy.

Yet, studies conducted in the United States suggest more similar patterns of cognitive influence across fluency and accuracy. In a sample of 201 children, Hecht et al. (2001) found that second-grade PP predicted fifth-grade MC accuracy and fluency as well as sight-WR and word attack accuracy (while controlling for oral vocabulary). In Geary (2011), where the focus was accuracy in 177 children, WM predicted WR and MC across grades 1 to 5, while PP and RAN letters were unique to the WR outcome and RAN numerals were unique to the MC outcome.

In 2016, Fuchs et al. extended earlier studies by modeling the role of domain-general and core cognitive predictors while considering the reciprocal effects of early mathematics on WR outcomes and early reading on MC outcomes. In 747 children, first-grade WR directly related to third-grade MC accuracy, with additional indirect effect via second-grade arithmetic retrieval. Meanwhile, first-grade mathematics effects on third-grade WR accuracy were largely indirect via arithmetic retrieval. Visuospatial memory,

attentive behavior, nonverbal reasoning, and RAN contributed to both outcomes indirectly via arithmetic retrieval. Unique direct predictors were language comprehension (i.e., understanding the semantics and syntax of oral language) for WR and WM for MC.

More recent studies employed more targeted methodological approaches for understanding the role of shared cognitive processes in WR and MC development by examining predictors of shared variance between WR and MC. We describe four studies representative of this literature. With 233 second-grade children, Child et al. (2019) compared zero-order correlations between WR and MC to partial correlations controlling variance associated with one cognitive process at a time. Relations between cognitive processes and overlap were similar for fluency and accuracy outcomes. PP and WM contributed to overlap substantially; processing speed and numerosity to a lesser extent.

Using a similar approach, Cirino et al. (2018) followed 193 children from kindergarten through first grade. Zero-order correlations between WR and MC dropped substantially when partial correlations between WR and MC controlled for the study's full set of measured cognitive processes: 75% reduction for accuracy and 84% for fluency. Partial correlations again suggested similar patterns for accuracy and fluency. PP, RAN, and symbolic naming each produced a large percentage reduction. Rote counting, symbolic comparison, oral language comprehension, nonverbal reasoning, and attentive behavior each produced a moderate percentage reduction. Smaller contributions occurred from WM and counting knowledge and processing speed.

To quantify contributions in a more integrated manner, Koponen et al. (2020) employed structural equation modeling. A sample of 200 Finnish children was followed from the spring of first grade to the fall of second grade. The exclusive focus was fluency outcomes because WR and MC accuracy was assumed at the study's endpoint based on the language's transparent orthography and the country's explicit instructional methods and well-functioning instructional prevention systems. First-order latent factors of WR and MC were created; then, a second-order latent Comorbidity factor was used to model covariance between WR and MC. Core cognitive processes were entered as predictors of the Comorbidity factor, followed by domain-general cognitive processes as direct effects and indirect effects via core processes. Serial retrieval fluency (a latent factor across RAN and VC) accounted for a major share of variance between WR and MC, with additional contributions from other core processes: PP, number comparison, and number writing. Domain-general processes (WM, articulation speed, processing speed) explained half the variance in serial retrieval fluency, and processing speed was directly related to the Comorbidity factor. WM, articulation speed, verbal short-term memory, and processing speed as well as visuospatial memory were related indirectly via serial retrieval fluency.

Using a similar approach to follow Finnish children from the end of first through seventh grade, Korpipää et al. (2017) found that most of the covariation between grades was time-invariant and could be predicted by RAN, VC, letter knowledge, WM, and nonverbal reasoning. The time-specific portion of first-grade covariation between reading and arithmetic was predicted by PP, letter knowledge, and counting.

Summary of Prior Research

Across studies, we draw two major conclusions about potential connections between cognitive processes and early WR and MC difficulty. First, relations may differ depending on the orthographic transparency of the language in which learning to read occurs. Most relevant to the present study, the pattern of potentially important cognitive processes appears more alike than different for accuracy and fluency outcomes when primary-grade children are operating in an opaque orthography (like English). This is reflected in Child et al. (2019), Cirino et al. (2018), and Hecht et al. (2001).

Second, individual differences in WR and MC and in shared variance between WR and MC are associated with a combination of core and domain-general cognitive abilities. The distinction between core and domain-general cognition permeates the literature on comorbid WR and MC development. In terms of core cognitive processes, the importance of fluent retrieval of symbolic stimuli (letters and numerals) and the ability to recognize and manipulate sound segments in spoken words seems clear. This is most consistently represented in prior work by RAN, VC, and PP tasks. More complex, domain-general abilities that recur as potentially salient are attentive behavior, WM, language comprehension, nonverbal reasoning, and visuospatial memory.

Present Study

The present study follows Koponen et al.'s (2020) reliance on structural equation modeling to predict a latent Comorbidity factor and describe core cognitive and domain-general cognitive processes associated with comorbid difficulty across WR and MC. The context provided in the present study differs from previous studies investigating predictors of comorbid WR and MC difficulties in the following ways. First, we focused on the start of first grade, a critical juncture in formal schooling when reading (Shaywitz, 1998), mathematics (Duncan et al., 2007), and comorbid difficulty (Koponen et al., 2018; Landerl & Moll, 2010) may become intractable without responsively timed intervention. Second, our sample comprised children with

early comorbid WR and MC difficulty. By contrast, prior work has been conducted with representative samples. (Child et al. [2019] included unselected children as well as some selected for MC but not WR difficulty.) Although reading and mathematics skills and most predictors are dimensional, it is possible that qualitative differences in associations with covariance exist in children with comorbid difficulty. Third, we combined accuracy and fluency when modeling WR and MC performance, given our focus on students with comorbid WR and MC difficulty operating in an opaque orthography. Note that we also accounted for dependency at the teacher level in our statistical modeling and focused on concurrent associations.

Predictive relations indicate whether individual differences in cognitive processes at an earlier stage facilitate later outcomes. Finding a similar pattern for concurrent associations at the start of first grade, especially in a sample of children selected for substantial delays in WR and MC, would lend credence to the idea that cognitive targets are rich in opportunity for coordinated first-grade intervention. Divergent findings would suggest the need for an alternative approach.

The hope is that by focusing on children with severe delays in WR and MC at the start of first grade and accounting for shared cognitive processes across WR and MC, findings provide insight into opportunities for designing first-grade intervention to strengthen WR and MC in a coordinated fashion. This is an important goal in the context of the scheduling challenges and intervention costs, which mitigate against schools providing more than one intervention to the same child. Unfortunately, reading intervention often takes priority over mathematics intervention, leaving many of these children underserved.

With a focus on recurring effects in prior work (RAN, PP, and VC as core processes; WM, attentive behavior, oral language, and nonverbal reasoning as domain-general processes), we hypothesized that the core cognitive processes together directly account for a substantial portion of shared variance between WR and MC and that effects of domain-general processes largely accrue indirectly via these core cognitive processes. That is, we expected domain-general processes to be associated with core cognitive processes and expected stronger core processes to in turn explain covariance between WR and MC. This direction of effects from domain-general to core processes (rather than from core to domain-general processes) is established in the literature. It reflects two big ideas. The first is that the development of core cognitive processes, involving stimuli more proximal to the predicted learning targets, emerges in typically developing children during preschool. The second is that the development of core cognitive processes depends on domain-general cognitive abilities present at younger ages (Chu et al., 2016).

Even so, in interpreting results, we refrain from causal inference. We instead consider concurrent relations between cognitive processes and shared covariance in WR and MC in terms of opportunities for structuring innovative interventions to strengthen WR and MC in a coordinated fashion and with efficiency by accounting for shared cognitive processes. The long-term hope is that coordinated intervention can achieve similarly strong outcomes on WR and MC as single-focus intervention conventionally requires but without double the intervention time.

Method

Participants

We conducted this study in accordance with our university-approved institutional review board (IRB) protocol (this IRB is charged with ensuring compliance with ethical and legal standards) and prior to inclusion in study, children provided assent, their parents or legal guardians provided parental consent, and their teachers provided consent.

The sample was drawn from a large, diverse, urban, and suburban county-wide school district in the southeastern United States. To identify a sample with comorbid WR and MC difficulty, we relied on a multi-stage screening procedure. In Stage 1, 1,651 children with parent consent and self-assent completed the *First-Grade Test of Mathematics Computation* (Fuchs et al., 1990; see the "Measures" section for the description of this and other measures) in large groups. An established cut score (<25th percentile) was applied to identify a pool of 513 children with low MC. In Stage 2, teachers excluded 47 children with or suspected of having a disability other than a learning disability or with insufficient English to assume valid test scores or with schedules that precluded participation. In Stage 3, the remaining 466 children were individually tested on *Word-Identification Fluency* (WIF; Fuchs et al., 2004) and the *Two-Subtest Wechsler Abbreviated Scale of Intelligence* (WASI; Wechsler, 2011); 119 were excluded due to scores at or above the 25th percentile on WIF (to ensure low WR), and 38 children were excluded because they did not score above 79 on at least 1 WASI subtest (to ensure cognitive ability within the broadly average range). Four children moved before completing the assessment battery. From the remaining 305 children, we randomly selected 234 (with 88 teachers in 20 schools) to meet our recruitment goal.

The sample's mean age was 6.50 years ($SD = 0.31$); 54.4% were female; 57.4% were from households with economic disadvantage (EDIS; i.e., at least one form of state financial aid, certified by the state to the school district); 32.3% were African American, 26.0% White non-Hispanic, 35.0% White Hispanic, and 7.7% other. Thirty-five percent qualified for English services; 9.8% for special education. On the *Wide Range Achievement Test—Reading* (4th ed.; WRAT4; Wilkinson & Robertson, 2006; see the "Measures" section), mean standard scores were 72.66 ($SD = 9.12$; 4th percentile) on Reading (WRAT4-WR) and 81.27 ($SD =$

Table 1. Correlations Among Manifest Variables.

Variable	1	2	3	4	5	6	7	8	9	10	11	12	13	14
1. WRAT–Read														
2. WJ–WID	.69													
3. SWE	.57	.74												
4. WRAT–Math	.36	.43	.41											
5. WJ–CAL	.26	.38	.34	.58										
6. Arith Comb	.23	.24	.18	.30	.30									
7. Double–Digits Cal	.10	.10	.05	.34	.37	.42								
8. PP	.34	.52	.39	.53	.40	.12	.15							
9. RAN	−.54	−.53	−.46	−.36	−.29	−.23	−.12	−.32						
10. VC	.52	.43	.43	.52	.39	.23	.21	.41	−.37					
11. WM	.23	.27	.27	.47	.41	.26	.19	.43	−.27	.31				
12. Language	.13	.15	.12	.27	.21	.09	.10	.49	−.04	.31	.34			
13. NVR	.05	.08	.04	.21	.22	.17	.33	.14	.04	.19	.20	.02		
14. Attention	.06	.17	.16	.19	.21	.16	.06	.08	−.15	.09	.22	.05	.09	
Mean	13.43	14.98	6.23	11.39	6.60	2.22	0.54	8.06	57.33	3.26	5.27	9.90	5.75	30.69
SD	3.69	4.15	4.60	2.61	4.68	2.44	1.33	3.98	29.55	2.01	2.33	5.05	2.53	7.59

Note. WRAT-Reading = *Wide Range Achievement Test—Reading*; WJ-WID = *Woodcock Johnson—Word Identification*; SWE = *Test of Word Reading Efficiency—Sight Word Efficiency*; WRAT-Math = *Wide Range Achievement Test—Mathematics Computation*; WJ-CAL = *Woodcock Johnson—Calculation*; Arith Comb = *Arithmetic Combinations Fluency*; Double-Digit Cal = *Double-Digit Calculations Fluency*; PP = phonological processing; RAN = rapid automatized naming; VC = verbal counting; WM = working memory; Language = language comprehension; NVR = nonverbal reasoning (*Wechsler Abbreviated Scale of Intelligence—Matrix Reasoning*); Attention = attentive behavior (*SWAN Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder Symptoms and Normal Behavior Scale*).

10.71; 10th percentile) on Mathematics Calculations (WRAT4-MC). On average, children named 12.23 (*SD* = 3.15) of 15 letters and read 1.21 (*SD* = 1.23) of 55 words. They answered 10.30 (*SD* = 2.18) of 15 early numerical items and 1.09 (*SD* = 1.00) of 40 MC problems correctly. See Supplemental File Table 1 for frequency counts of items by subtest.

Measures

Study-Entry Screening Measures. With *First-Grade Computation Fluency* (Fuchs et al., 1990), children have 2 min to complete 25 computation items. Test–retest reliability at this age is .87. With *Word Identification Fluency* (Fuchs et al., 2004), students have 1 min to read 50 words randomly sampled from 100 high-frequency pre-primer, primer, and first-grade words. If a student finishes before 1 min, the score is prorated. Scores are averaged across two alternate forms. Test–retest reliability at this age is .94. *Wechsler Abbreviated Scale of Intelligence* (WASI; Wechsler, 2011) is a two-subtest measure of general cognitive ability, comprising *Vocabulary* and *Matrix Reasoning* subtests (reliability > .92). *Vocabulary* assesses expressive vocabulary, verbal knowledge, memory, learning ability, and crystallized and general intelligence. Students identify pictures and define words. *Matrix Reasoning* measures nonverbal fluid reasoning and general intelligence. Children complete matrices with missing pieces. Sample-based α was .81 and .84, respectively.

WR Measures. With *WRAT4-WR* (Wilkinson & Robertson, 2006), children name letters and read words of increasing difficulty without a time limit. Sample-based α was .88. With *Woodcock-Johnson IV Tests of Achievement–Letter-Word Identification* (WJIV-LWID; Schrank et al., 2014), children name letters and read words of increasing difficulty without a time limit. Sample-based α was .87. With *Test of Word Reading Efficiency-2–Sight-Word Efficiency* (SWE; Torgesen et al., 2012), children have 45 s to read words of increasing difficulty. Sample-based α was .87.

MC Measures. *WRAT-MC* (Wilkinson & Robertson, 2006) includes an oral section assessing early numerical competencies (counting; naming, representing, comparing numerals; solving simple word problems) and a written portion in which 90% of items require MC. Sample-based α was .89. With *WJIV–Calculations* (Schrank et al., 2014), children complete MC items of increasing difficulty. Sample-based α was .82. With *Arithmetic Combinations Fluency* (Fuchs et al., 2003), children have 1 min to write answers to 25 addition problems and 1 min to write answers to 25 subtraction problems (sums and minuends 5–12). The score is number correct across 50 items. Sample-based α was .91. With *Double-Digit Calculations Fluency* (Fuchs et al., 2003), children have 10 min to write answers to 20 addition and 20 subtraction problems with and without regrouping. Sample-based α was .84.

Measures of Core Cognitive Processes. PP was indexed with two measures. With the *Comprehensive Test of Phonological Processing-Elision* (CTOPP-2; Wagner et al., 2013), children say a word with a constituent part removed. Sample-based α was .82. With *CTOPP2-Sound Matching*, children select words with the same initial or final sound. Sample-based α was .80.

RAN was indexed with CTOPP-2 (Wagner et al., 2013): Digits and Numerals. After completing three practice trials that involve naming letters and numerals as quickly as possible, the tester displays a 5 × 10 array of letters; the child names them as quickly as possible. The tester uses a stopwatch to index seconds elapsed. Then digits are completed. Test–retest reliability with a subsample of 70 children was .81.

VC was indexed with the *Number Sequences Test* (Salonen et al., 1994). The child counts as quickly as possible: forward from 1 to 31, forward from 6 to 13, backward from 12 to 7, and backward from 23 to 1. If the child is silent for 5 s after the tester delivers the prompt to start, the tester proceeds to the next task. For each task, correct performance earns two points; completion with fewer than three errors, one point. Sample-based α was .83.

Measures of Domain-General Cognitive Processes. WM (i.e., storage and manipulation of a limited amount of information over a short amount of time; Cowan, 2014) was indexed with three measures. With *Working Memory Test Battery-Children* (WMTB-C)-*Mazes Memory* (Pickering & Gathercole, 2001), the tester presents a maze with more than one solution and a picture of an identical maze with a path revealing one solution. The picture is removed; the child duplicates the shown path. (Mazes Memory was originally planned as a measure of visuospatial memory; however, preliminary measurement models indicated superior model fit as a measure of WM; see Miyake et al., 2001). With *Automated Working Memory Assessment* (*AWMA*; Alloway, 2007)-*Listening Recall*, the child decides if sentences are true and then recalls the last word of sentences at the end of the series. With *AWMA-Counting Recall*, the child counts circles in an array and recalls the tallies at the end of the series. For each measure, task trials begin with one item; at each block, the number of walls, words, or counts increases by one until the child cannot fail to recall three of six trials in a block. The score is number of correct trials. Sample-based α for the 3 tasks was .82 to .85.

Oral language comprehension (i.e., understanding of the syntax and semantics of oral language) was indexed with two measures. With *Woodcock Diagnostic Reading Battery-Listening Comprehension* (Woodcock, 1997), children supply words missing from the end of sentences or passages, progressing from simple verbal analogies and associations to discerning implications. With *WASI Vocabulary* (Wechsler, 2011), students identify pictures and define words. Sample-based α for the two measures was .80 and .81.

Nonverbal reasoning (i.e., problem-solving using pictures and diagrams) was measured with *Matrix Reasoning*, in which children complete matrices with missing pieces. Sample-based α was .83. To index attentive behavior (i.e., being alert and actively paying attention to stimuli), we used the *Strengths and Weaknesses of Attention-Deficit/Hyperactivity Disorder Symptoms and Normal Behavior Scale* (*SWAN*; Bussing et al., 2008; Swanson et al., 2012), in which teachers rate nine attention items from the *Diagnostic and Statistical Manual of Mental Disorders* for Attention Deficit Hyperactivity Disorder, each on a 7-point scale. The score is the sum of the ratings. Sample-based α was .97.

Procedure

In August, we screened children for study entry. In September to October, we conducted testing individually and in small groups. Testing sessions were audio-recorded; 15% of recordings were randomly selected (stratified by the tester) and checked for accuracy by an independent scorer. Agreement exceeded 99%.

Transparency and Openness

This report provides the basis for participant exclusions and describes data manipulations and analyses. This report's data and data codebook are available at https://doi.org/10.33009/ldbase.1690146207.e3c1. The design for the present analyses was not preregistered.

Analyses and Results

Correlations among the manifest variables, and descriptive statistics for each, are provided in Table 1. Correlations among the latent variables and predictors are presented in Table 2. Intraclass correlations (ICCs) for reading and mathematics scores were .00 to .08 at the teacher level. We adjusted standard error estimates to account for teacher-level dependencies using the Type = Complex option with MLR estimator in M*plus* 7.4 (Muthén & Muthén, 1998–2013). We fit a series of confirmatory factor analyses and structural models in M*plus* 7.4 and evaluated the adequacy of the model using multiple fit indices: chi-square, comparative fit index (CFI), Tucker–Lewis index (TLI), root mean square error of approximation (RMSEA), and standardized root mean squared residuals (SRMR). We followed Hu and Bentler's (1999) guidelines for evaluating the adequacy of fit: CFI and TLI values equal to or greater than 0.95, RMSEA values with the upper boundary at or less than 0.06, and SRMR equal to or less than 0.08.

We began by fitting a two-correlated latent factor model of reading, estimated from the three WR measures, and mathematics, estimated from the four MC measures. The initial model fit (RMSEA = 0.10; CFI = 0.95; TLI = 0.92; SRMR = 0.06) was improved by estimating residual

Table 2. Estimated Correlation for the Latent Variables and Predictors.

Variable	1	2	3	4	5	6	7	8	9	10
1. WR (latent)										
2. MC (latent)	.55									
3. Comorbidity (latent)	.75	.73								
4. PP	.52	.54	.73							
5. RAN	−.63	−.43	−.59	−.35						
6. VC	.56	.58	.79	.43	−.38					
7. WM	.43	.44	.60	.44	−.31	.31				
8. Language	.23	.24	.32	.49	−.06	.31	.34			
9. NVR	.17	.18	.25	.14	.01	.19	.20	.02		
10. Attention	.20	.20	.28	.07	−.16	.09	.22	.05	.09	

Note. WR = word reading; MC = mathematics computation; PP = phonological processing; RAN = rapid automatized naming; VC = verbal counting; WM = working memory; Language = language comprehension; NVR = nonverbal reasoning; Attention = attentive behavior.

covariance between the two MC fluency measures (RMSEA = 0.07, CFI = 0.98, TLI = 0.97, and SRMR = 0.04). We then fit a higher-order factor model, comprising the first-order factor in each domain (WR; MC) and a second-order Comorbidity factor indexing covariance between the first-order factors. In this model, latent residuals indicate the unique variation of first-order factors (WR; MC) that is not explained by the second-order factor (the Comorbidity factor) or the residuals of observed variables. To identify the model, we constrained factor loadings of the Comorbidity factor to be equal between WR and MC. As in Figure 1, although standardized factor loadings for the two mathematics fluency measures were lower than for other mathematics measures, the final measurement model revealed a good fit to the data (RMSEA = 0.07; CFI = 0.98; TLI = 0.97; SRMR = 0.04).

Relations Between Core Cognitive Processes and the Comorbidity Factor

To assess the relations between core cognitive processes and the Comorbidity factor, the model included RAN, PP, and VC. We also included EDIS as a covariate to reflect its inclusion in the literature. Note that the correlations among measures *within each construct* were not sufficiently high to warrant latent variables. When modeled as latent variables, very high correlations *between* VC and the Comorbidity factor (a correlation greater than 1) caused the Heywood case. Therefore, we represented each core process as a manifest variable (i.e., mean PP score, mean RAN score, mean VC score). This model showed an overall good fit to data (RMSEA = 0.08, CFI = 0.94; TLI = 0.92; SRMR = 0.05). Each core cognitive process was significantly related to the Comorbidity factor, with 92% of the variance explained (see Figure 2). For RAN, β = −.39 (*p* < .01); PP, β = .45 (*p* < .01); for VC, β = .42 (*p* < .01).

Relations Between Domain-General Cognitive Processes and the Comorbidity Factor

The next model included the four domain-general processes as direct effects and as indirect effects via each core cognitive process. With covariances among predictors freely estimated, the model demonstrated a good fit to data (RMSEA = 0.07; CFI = 0.93; TLI = 0.89; SRMR = 0.05), and the predictors explained 99% of the variance in the Comorbidity factor. In this model (see Figure 3), the direct effect of each core cognitive process on the Comorbidity factor (i.e., the *b*-path within indirect effects) remained significant (for PP β = .43, *p* < .01; RAN β = −.35, *p* = .03; VC β = .41, *p* < .01).

The four domain-general cognitive processes together explained an additional 7% of the variance in the Comorbidity factor. Direct relations with the Comorbidity factor were significant for WM and attention (β = .17, *p* < .01 and β = .13, *p* = .01, respectively). Indirect effects of domain-general influences via core cognitive processes on the Comorbidity factor (95% confidence intervals (CIs) using the bootstrapping method with 1,000 draws) were as follows. WM operated indirectly through RAN (β = .10, 95% CI [0.05, 0.17]), PP (β = .12, 95% CI [0.05, 0.20]), and VC (β = .08, 95% CI [0.03, 0.12]). Language operated indirectly through PP (β = .17, 95% CI [0.09, 0.24]) and VC (β = .10, 95% CI [0.05, 0.15]). Nonverbal reasoning operated indirectly through VC (β = .06, 95% CI [0.02, 0.11]).

Discussion

The purpose of this analysis was to describe the cognitive processes associated with comorbid difficulty between WR and MC at the start of first grade. We described these relations in a sample of children with substantial delays in both academic domains, at a time when formal schooling on WR and MC gains momentum. The goal was to provide insight for designing first-grade interventions to address children's WR and
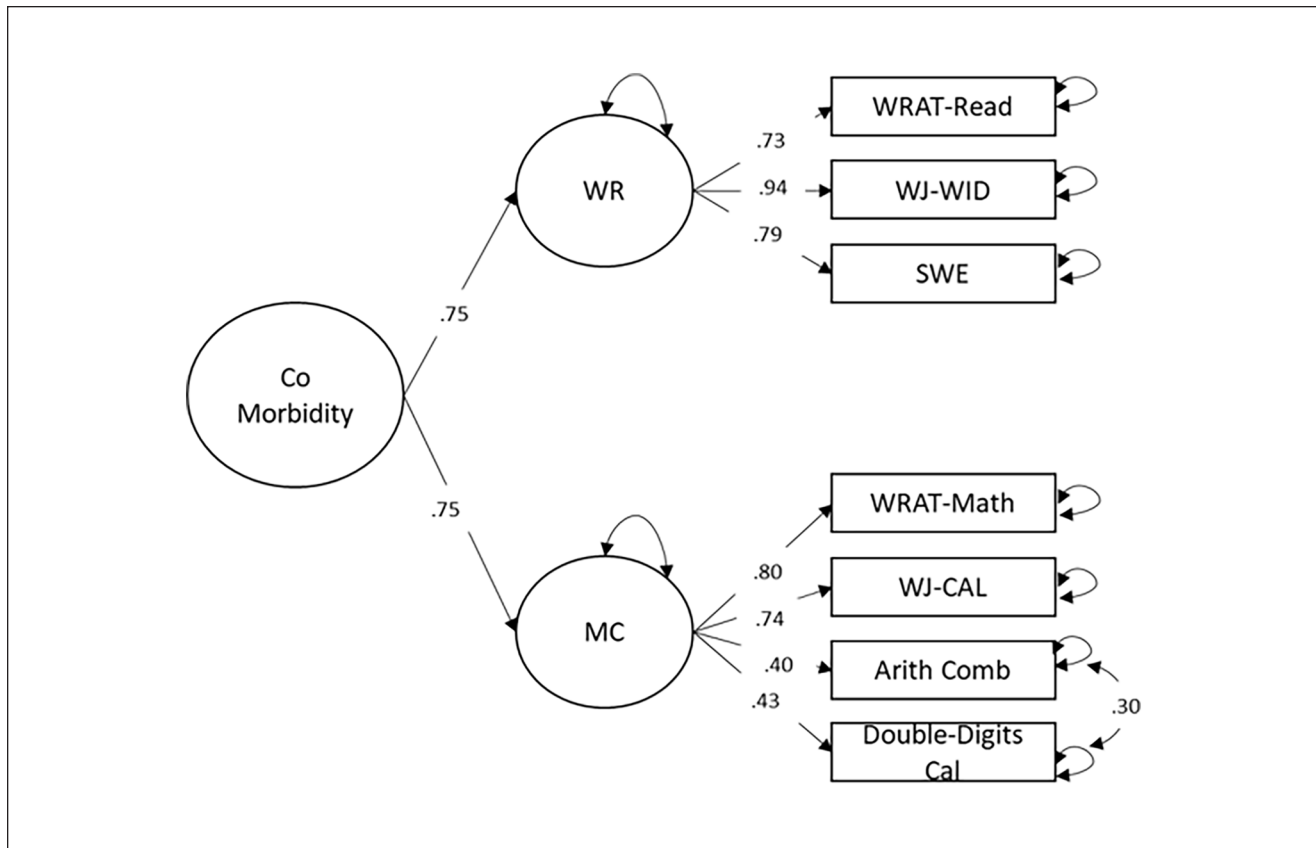
Figure 1. Measurement Model.

Note. WR = word reading; MC = mathematics computation; WRAT-Read = *Wide Range Achievement Test—Reading*; WJ-WID = *Woodcock Johnson—Word Identification*; SWE = *Test of Word Reading Efficiency—Sight Word Efficiency*; WRAT-Math = *Wide Range Achievement Test—Mathematics Computation*; WJ-CAL = *Woodcock Johnson—Calculation*; Arith Comb = *Arithmetic Combinations Fluency*; Double-Digits Cal = *Double-Digit Calculations Fluency.*

MC needs in a coordinated and efficient fashion by accounting for shared cognitive processes across WR and MC. In this discussion, we first consider relations between core cognitive processes and the Comorbidity factor used to model covariance between WR and MC (RQ 1). Next, we discuss relations involving domain-general cognitive processes (RQ 2). Then we identify study limitations, draw conclusions and contributions to the literature, explore implications for designing first-grade interventions, and discuss future research.

Relations Between Core Cognitive Processes and the Comorbidity Factor

Consistent with our first research question's hypothesis, the three core processes included in our analysis together accounted for a substantial portion of shared variance between WR and MC. Without domain-general processes in the model, PP, RAN, and VC explained 92% of the variance in the Comorbidity factor. After accounting for direct and indirect relations involving domain-general processes, the percentage of variance

explained by the core processes remained significant and similarly strong.

We indexed PP with tasks of phonological awareness, a signature core process in the development of early WR skill: Mapping the sound structure of language is foundational for decoding skill, which in turn provides the basis for WR (Catts et al., 2002; Grigorenko et al., 2020; Wagner & Torgesen, 1987). PP is also featured in the developmental MC literature: In supporting the counting routines (count all, count on, count backward, count difference) required for success with simple addition and subtraction (Kroesbergen et al., 2009), counting creates the foundation for automatic retrieval of answers to arithmetic problems (Zhang et al., 2014).

Accordingly, the relation between PP and the Comorbidity factor in our analysis was substantial: $\beta = .45$ without general processes in the model and .43 with those processes included. This finding lends support to prior work in which PP was foundational to co-development of WR and MC skills (e.g., Child et al., 2019; Cirino et al., 2018; Hecht et al., 2001; Koponen et al., 2020; Korpipää et al.,
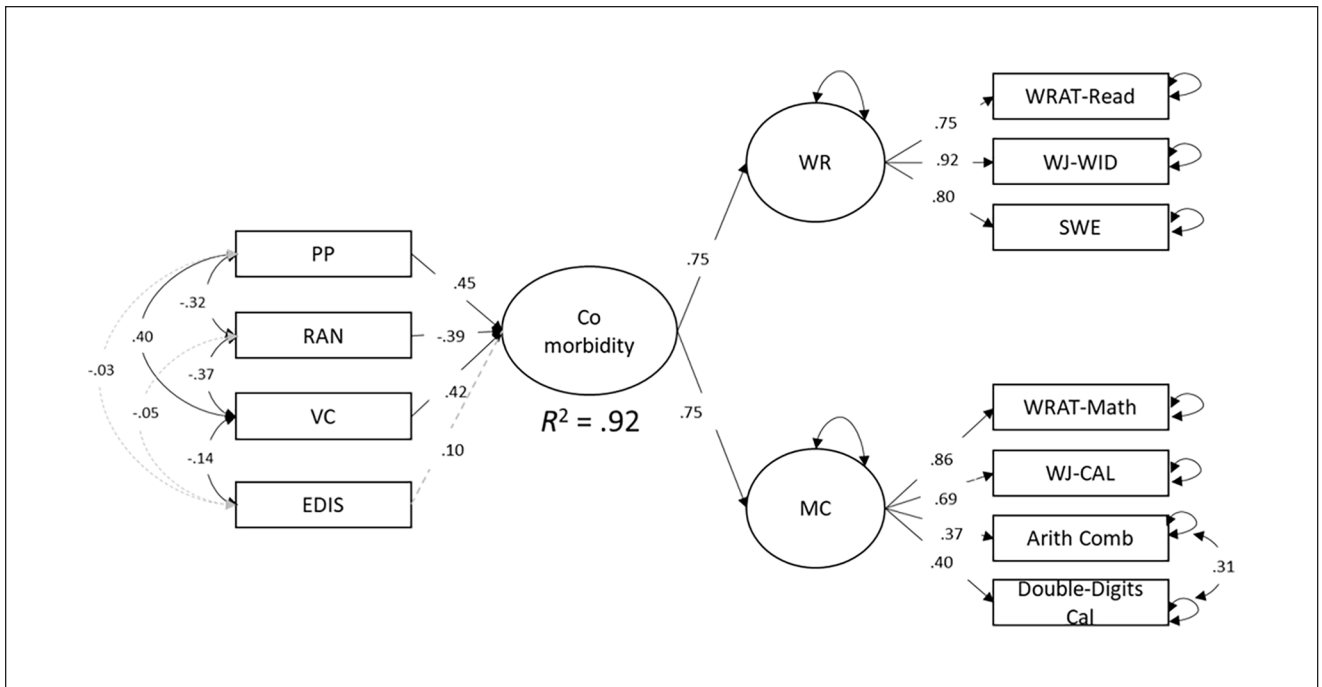
Figure 2. Lower-Level Cognitive Processes Model.
Note. WR = word reading; MC = mathematics computation; PP = phonological processing; RAN = rapid automatized naming; VC = verbal counting; EDIS = economically disadvantaged; WRAT-Read = *Wide Range Achievement Test—Reading*; WJ-WID = *Woodcock Johnson—Word Identification*; SWE = *Test of Word Reading Efficiency—Sight Word Efficiency*; WRAT-Math = *Wide Range Achievement Test—Mathematics Computation*; WJ-CAL = *Woodcock Johnson—Calculation*; Arith Comb = *Arithmetic Combinations Fluency*; Double-Digits Cal = *Double-Digit Calculations Fluency.*
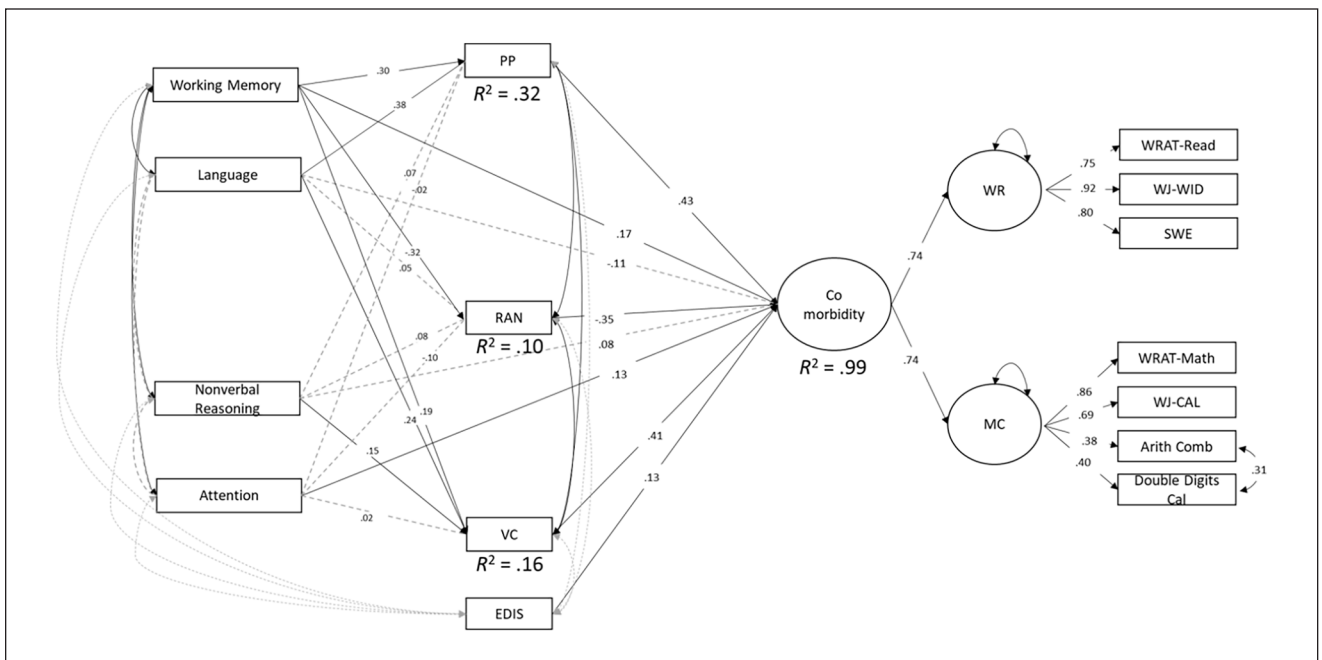

Figure 3. Relations With Comorbidity for Higher-Order Cognitive Processes: Direct Effects and Indirect Effects Via Lower-Order Cognitive Processes.
Note. WR = word reading; MC = mathematics computation; PP = phonological processing; RAN = rapid automatized naming; VC = verbal counting; EDIS = economically disadvantaged; WRAT-Read = *Wide Range Achievement Test—Reading*; WJ-WID = *Woodcock Johnson—Word Identification*; SWE = *Test of Word Reading Efficiency—Sight Word Efficiency*; WRAT-Math = *Wide Range Achievement Test—Mathematics Computation*; WJ-CAL = *Woodcock Johnson—Calculation*; Arith Comb = *Arithmetic Combinations Fluency*; Double-Digits Cal = *Double-Digit Calculations Fluency.*

2017), although the literature is not consistent on this point (Amland et al., 2021; Cirino et al., 2015; Fuchs et al., 2016; Geary, 2011; Koponen et al., 2007).

Despite that all three core processes involve the phonologic system, RAN, and VC tasks, in contrast to PP (i.e., phonological awareness), RAN and VC simultaneously involve speeded or serial memory performance demands. Evidence for unique relations with the Comorbidity factor for these two core processes, beyond phonological awareness, was clear. For RAN, β = −.39 without domain-general processes in the model and −.35 when included; for VC, β = .42 without domain-general processes in the model and .41 when included.

RAN and VC, with their simultaneous speeded or serial memory demands, may reflect the ability to form and fluently retrieve from memory arbitrary associations between the visual symbolic and phonologic forms. This is in keeping with Koponen et al.'s (2013) suggestion as to why performance on counting measures predicted both calculation and reading fluency and is consistent with Koponen et al.'s (2020) later demonstration that this ability is critical in the development of comorbid WR and MC difficulty.

As Ünal et al. (2023) explained, this finding may partially reflect the fact that learning in both domains depends on the same underlying brain systems and cognitive processes (also see Pennington, 2006), specifically the functional integrity of the hippocampal-dependent memory system, which during the early phases of learning engages prefrontal, parietal, and medial temporal areas (Qin et al., 2014). This system has been shown to be important in learning written words (Cherodath & Singh, 2015) and arithmetic facts (De Smedt et al., 2011; Qin et al., 2014). Thus, relations between WR and MC likely reflect underlying individual differences in the ease of this form of associative learning (Supekar et al., 2013).

The strength of VC's relation with the Comorbidity factor is especially interesting given its more transparent connection to MC than WR. The observed value of its relation with the Comorbidity factor (β = .41) was similar to its relation to RAN (β = −.35), even though RAN involves letters as well as numerals. A similarly strong relation for VC with the Comorbidity factor may reside with VC's engagement of a broader constellation of domain-general abilities, a point we discuss in the next section.

Relations Between Domain-General Cognitive Processes and the Comorbidity Factor

WM, oral language, nonverbal reasoning, and attentive behavior collectively explained an additional 7% of the variance in the Comorbidity factor. Consistent with our second research question's hypothesis, these associations largely accrued indirectly via the core cognitive processes.

This makes sense because, compared with domain-general cognitive processes, core cognitive processes are more proximal to WR and MC, with greater reliance on words, letters, and numerals (Cirino et al., 2018).

VC's relatively strong engagement of domain-general abilities is reflected in the triad of indirect effects that occurred via VC: for WM (β = .08), oral language (β = .10), and nonverbal reasoning (β = .06). The significant *a*-path coefficients associated with these indirect effects (.19, .24, and .15) reflect the idea that WM, oral language, and nonverbal reasoning, respectively, are associated with VC, which in turn is related to covariance between WR and MC.

In thinking about WM's indirect effect, we note that the VC score comprised four tasks presented in the same test session: counting as quickly as possible from 1 to 31, from 6 to 13, from 12 to 7, and from 23 to 1. These counting procedures and variations among them demand executive functions involving updating information, inhibiting prepotent responses, and shifting between tasks (Miyake et al., 2000). It is therefore not surprising that the effects of the study's complex span WM tasks operated indirectly via VC on the Comorbidity factor. A role for WM in comorbid WR and MC has been demonstrated frequently in prior work (Child et al., 2019; Fuchs et al., 2016; Geary, 2011; Koponen et al., 2020), but see Cirino et al. (2018).

At the same time, given the interplay between PP and VC (Kroesbergen et al., 2009; Zhang et al., 2014) and the naming demands involved in the VC tasks, it is not surprising that oral language's effects on the Comorbidity factor also operated indirectly via VC. Furthermore, given the role of nonverbal reasoning within early numerical competencies including counting (Chu et al., 2016; Homung et al., 2014) as well as VC's demands on the planning, inhibition, and attentional control required in nonverbal reasoning (e.g., Arán-Filippeti & Richaud, 2017; Homung et al., 2014), nonverbal reasoning's indirect effects on the Comorbidity factor via VC also make sense. Although the effect of nonverbal reasoning on co-developing WR and MC has previously been demonstrated (Cirino et al., 2018; Fuchs et al., 2016; Kroesbergen et al., 2009, but see Spencer et al., 2022), nonverbal reasoning has received less attention in the comorbid WR and MC literature than some other domain-general processes.

Meanwhile, WM's relations with the Comorbidity factor were pervasive. Indirect relations between WM occurred through each of the three core cognitive processes: β = .08 through VC; β = .10 through RAN; and β = .12 through PP. More impressively, with these indirect effects accounted for in the model, WM's direct effect on the Comorbidity factor was also significant and large (β = .17). This echoes Koponen et al. (2020), who identified WM as having the strongest unique contribution on the shared variance

between WR and MC fluency. In the present study, with WM's indirect and direct effects accounted for in the model, a direct relation between attentive behavior and the Comorbidity factor was also significant (β = .13).

The joint importance of WM and attentive behavior corroborates and extends Spencer et al. (2022), who investigated WR and MC outcomes rather than shared variance between them. Finding direct relations for WM and attentional control with respect to shared variance between WR and MC in a sample of children who begin first grade with comorbid delays highlights the central role of effortful problem-solving processes in the early stages of WR and MC. This finding reflects children's engagement in serial phonemic coding and assembly of letter sounds to decode words and serial recitation and integration of number–word quantities to derive sums, just as neurobiological findings reveal a role for executive functions in reading and mathematics (Arsalidou et al., 2018; Arsalidou & Taylor, 2011; Jobard et al., 2003; Wang et al., 2020). Also, Barnes et al. (2020) identified attention as a kindergarten marker for comorbid reading and mathematics performance.

Finally, an indirect relation between oral language comprehension and the Comorbidity factor also occurred through PP (β = .17). This was expected given that PP is a central feature of language comprising the functional properties of sound. Cirino et al. (2018) and Snowling et al. (2021) demonstrated a role for oral language in WR and MC, but most findings are for WR (Fuchs et al., 2016; Ouellette, 2006) or MC (Homung et al., 2014; LeFevre et al., 2010). We located no prior investigation conducted in the context of shared variance between WR and MC. Koponen et al. (2020), who identified a role for PP in explaining shared variance, did not address the broader role of oral language capacity.

Limitations, Conclusions, Implications for Coordinated Intervention, and Future Research

The present analysis, while including a relatively comprehensive set of cognitive processes, did not include a measure of visuospatial memory or processing speed. With respect to visuospatial memory, the mazes memory measure was originally planned as such but, as noted, preliminary models indicated superior model fit as a measure of WM. This is consistent with Miyake et al. (2001). We did not include a measure of processing speed because findings for its contribution are mixed. Also, as Child et al. (2019) explained, processing speed is an ill-defined construct in the relevant literature, with increased measurement complexity appearing to increase its correlations and thus complicating the interpretation of its relation. Readers should also note that the relation between WR and mathematics calculations and the engagement of domain-general and

core cognitive processes at first grade and among children selected for low performance in both domains may differ at higher grades (see Cirino et al., 2024) and for unselected samples.

These limitations notwithstanding, our major conclusions are as follows. Findings converge with prior related studies by corroborating roles for a highly similar set of cognitive processes associated with WR and MC development. This includes roles for PP and oral language in building the platform needed to achieve co-occurring literacy and mathematics competence. It also includes associative learning's parallel role in supporting these outcomes, as suggested by RAN's and VC's relations with comorbidity. And it highlights the importance of the WM, attentional control, and nonverbal reasoning involved in the procedurally demanding problem-solving processes foundational to the co-emergence of WR and MC competence.

The present analysis extends prior work by demonstrating concurrent relations between these cognitive processes and the Comorbidity factor at the start of first grade, a critical juncture in formal schooling when reading (Shaywitz, 1998), mathematics (Duncan et al., 2007), and comorbid difficulty (Koponen et al., 2018; Landerl & Moll, 2010) may become intractable without responsively timed, multifaceted intervention. Our study also extends generalizations by demonstrating these relations in children selected for early comorbid WR and MC difficulty; by modeling WR and MC performance across accuracy and fluency; and by accounting for the statistical dependency at the teacher level.

By focusing on the cognitive processes associated with shared variance between WR and MC in children with comorbid difficulty at a crucial time, the hope is that by accounting for shared cognitive processes across WR and MC as intervention begins, findings provide direction for designing first-grade interventions in ways that strengthen performance in both academic domains in a coordinated fashion and with efficiency. This is an important goal given scheduling challenges and intervention costs as schools face the need to provide more than one intervention to the same child. This often leaves many of these children underserved. Finding a similar pattern for concurrent associations at the start of first grade, especially in a sample of children selected for substantial delays in WR and MC, lends credence to the idea that cognitive targets are rich in opportunity for coordinated first-grade intervention.

In considering potential directions, we make two evidence-based assumptions. First, because cognitive correlates of shared variance between WR and MC reflect recruitment of some of the same cognitive processes and brain systems, our findings suggest potential for mutualistic (bi-directional) relations between cognitive processes and academic performance (Fuchs et al., 2022; Peng & Kievit,

2020), and we therefore hypothesize that improved WR strengthens MC performance and vice versa.

Second, we assume that only a subset of the cognitive sources contributing to shared variance are potentially productive as intervention targets. In selecting a subset of cognitive processes as potential targets, we apply four criteria: evidence of malleability, evidence of transfer from cognitive training to reading outcomes, evidence of transfer from cognitive training to mathematics outcomes, and potential for embedding training in parallel ways on that process within WR and MC instruction (rather than providing decontextualized cognitive training). Embedding seems preferable over general cognitive training to minimize the loss of WR and MC instructional time and to address the transfer challenges students with learning difficulties often experience (National Research Council, 2000).

Given these considerations, coordinated treatment on PP (Bus & van IJzendoorn, 1999; De Smedt et al., 2010; Shanahan et al., 2008), WM (Peng, 2023; Peng & Fuchs, 2017), and attentional control (Ursache et al., 2012; Welsh et al., 2010) seems promising. With respect to PP, intervention may benefit from coordinated timing on a learning progression involving initial focus on sounds within individual letters and numerals, with gradual increases in the length and complexity of letters and numeral strings, sound units within words, and counting strategies for combining quantities and finding differences.

In terms of attentional control, a coordinated behavior management system for supporting self-regulation, perseverance, and hard work in the face of challenge and reinforcing accurate work may create synergy across WR and MC learning. Efficiency in intervention time is derived from the use of the same on-task behavioral strategies; the same self-monitoring behavioral system to support goal setting and accurate work; parallel focal activities for practice in and outside of intervention sessions; and integrated WR and MC homework.

In a similar vein, WM training tasks may be incorporated in parallel ways within decoding and counting-based arithmetic problem-solving. As argued by Peng (2023), such WM training tasks might link attentional control with the use of long-term memory through retrieval practice and be designed with meta-cognitive supports involving strategy to facilitate transfer across WR and MC. For example, learning may benefit from explicit discussions with children, supported by vignettes, about how attending to the sound structure of letters and numerals within WR and MC, how exercising WM strategically, and how exerting strong attention and motivation during intervention contributes to stronger learning in both academic areas.

Additional synergies and efficiencies may accrue via parallel activities to accelerate learning. This includes similar mnemonics to support associative learning; similar activities to encourage the use of personal confidence levels and attentional control to judge whether retrieval or analysis is preferred for a given task; fluency-building activities across WR and MC that are structured in parallel ways; and similar meaning-building strategies across domains for identifying main ideas, managing irrelevant information, and making inferences in text and word problems; and including games that mix letter–sound and numeral–number associations and mix decoding and counting demands.

A comprehensive study testing the effects of coordinated intervention would examine whether coordinated first-grade intervention outperforms a control group. Importantly, however, it would also test whether effects are non-inferior on WR outcomes to a same-duration condition that provides validated WR intervention and non-inferior on MC outcomes to a same-duration condition that provides validated MC intervention. Such a comprehensive study would further assess whether engagement of targeted cognitive processes contributes to outcomes by testing whether improved targeted cognitive processes mediate intervention effects on WR and MC outcomes and whether improved WR mediates effects on MC outcomes and vice versa. Finally, such a study would assess whether cognitive processes not addressed during coordinated intervention moderate effects of coordinated treatment or whether coordinated treatment compensates for non-targeted cognitive processes in first-grade children.

Declaration of Conflicting Interests

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Supplemental Material

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